Introduction

The split post dielectric resonator (SPDR) provides an accurate technique for measuring the complex permittivity of dielectric and ferrite substrates and thin films at a single frequency point in the frequency range of 1 to 20 GHz. Besides the SPDR fixture, a vector network analyzer such as the PNA or PNA-L and software package 85071E option 300, are required for the measurement. The measurement is automatic and easy to perform.

Comparison With Other Methods

The microwave methods of measuring the dielectric properties can be divided into two main categories [1]:

• transmission-reflection (e.g. measuring in coaxial line, waveguide, and free-space, or with open ended coaxial line) and
• resonance methods

The SPDR measurement technique is one type of resonance method. The transmission-reflection methods offer swept measurement at “any” point in the frequency range over which they operate, while the resonance methods use a single frequency (or, at most, a few frequency points for different modes).

Resonators and cavities offer the highest available accuracy for measurements of real permittivity, and allow for measurements of very low loss materials that can not be measured with other techniques. A measurement at a discrete frequency point(s) should be adequate, because lossless materials are nearly nondispersive. This means that their dielectric constant and loss tangent will stay constant over a range of frequencies.

The construction of the SPDR uses new, low loss dielectric materials which make it possible to build resonators having higher Q-factors and better thermal stability than traditional all-metal cavities [2,3]. The main advantages of the SPDR are:

• superior accuracy compared to transmission-reflection methods
• ability to measure low loss materials (lower loss materials that cannot be measured with the transmission-reflection technique)
• convenient, fast, and nondestructive measurement of substrates, printed circuit boards, and even thin films
This method is nondestructive, as long as the substrate can fit in the SPDR, because no special sample preparation is needed. The electric field in the resonator is parallel to the surface of the sample as shown on Figure 1. The main sample requirements are: two strictly parallel faces, the thickness of the sample in Figure 1, must be less than the fixture air gap $h_G$ (see Figure 4), and the sample must have enough area to cover the inside of the fixture. The air gap between the sample and the dielectric resonator (see Figure 4) does not affect the accuracy of the measurement. The sample may have a rectangular or round shape as shown in Figure 1. For easy handling of the sample it is recommended that the sample area dimension $L$ be bigger than the dimension of the minimum measurable area $l$ (or active area of the fixture).

Figure 1. Sample geometries

The required thickness of the sample also depends on the dielectric constant $\varepsilon_r'$ of the material. Materials with high dielectric constants must have less thickness. Figure 2 shows the typical resonant frequency $f$ versus permittivity $\varepsilon_r'$ in the case of a 10 GHz SPDR. For this fixture, if the permittivity of the sample $\varepsilon_r'$ is less than 10, the maximum sample thickness must be smaller than the fixture gap thickness $h_G$. At the same time, the sample must also be thick enough to create enough frequency shift to be easily measured. If the sample permittivity $\varepsilon_r'$ is greater than 10, the sample thickness may have to be reduced to keep the frequency shift within the recommended range. The thickness should be chosen from Figure 2, knowing that the frequency should not be much smaller than 8.5 GHz.
The fixture air gap $h_G$ and the active area dimension $l$ of the fixture depend on the operating frequency $f$ of the resonator. Table 1 shows approximate values of these dimensions for resonators operating at different frequencies. The sample dimension $L$ should be less than $L_f$, which is the maximal dimension that the fixture can accommodate.

**Table 1. Sample related dimensions of SPDR fixtures for different frequencies**

<table>
<thead>
<tr>
<th>$f$, GHz</th>
<th>$h_G$, mm</th>
<th>$l$, mm</th>
<th>$L_f$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>130</td>
<td>200</td>
</tr>
<tr>
<td>3.2</td>
<td>3.3</td>
<td>60</td>
<td>$\leq 150^*$</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>40</td>
<td>$\leq 150^*$</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>25</td>
<td>$\leq 150^*$</td>
</tr>
<tr>
<td>15</td>
<td>0.8</td>
<td>17</td>
<td>$\leq 100^*$</td>
</tr>
<tr>
<td>20</td>
<td>0.6</td>
<td>10</td>
<td>$\leq 100^*$</td>
</tr>
</tbody>
</table>

* The fixture can be ordered with $L_f$ dimension up to the indicated value but is recommended to be less, if there is no special need.

Figure 2. Typical resonant frequency versus permittivity for 10 GHz split post resonator
The SPDR technique can also be used for measuring thin films. Figure 3 shows the typical resonance frequencies $f$ versus the permittivity for a 10 GHz resonator. If the film is deposited on a substrate, the resonant frequency shift due to presence of the thin film is very similar to Figure 3 (differences are about 1 to 2 percent). To separate the frequency shift of the film from the overall frequency shift of the substrate and the film, the substrate alone should be measured initially (without film).

![Figure 3. Resonant frequency versus permittivity of a thin film deposited on artificial substrate with $\varepsilon'_r=1$ for 10 GHz split post resonator](image)

Permittivity and loss tangent of thin films deposited on substrates having a diameter > 20 mm can be evaluated directly with systematic 1 to 2 percent error using the same program as for uniform dielectrics. In this case one has to measure the empty resonator ($f_{01}, Q_{01}$), empty resonator with a substrate only ($f_s, Q_s$), and, after film deposition, repeat the measurement of the empty resonator ($f_{02}, Q_{02}$) and resonator with film and substrate ($f_{2}, Q_{2}$). The film should face down during this measurement.

When measuring films alone (not on a substrate), it is possible to stack them. The structure of the field is such that possible air gaps between the films will not affect the measurement. Results from measurements of stacked polymer films are independent of the number of films (see Table 2), which proves that the split dielectric resonator method is not sensitive to the presence of air gaps between the stacked films.

<table>
<thead>
<tr>
<th>$f$, MHz</th>
<th>Q</th>
<th>$h$, mm</th>
<th>$\varepsilon'_r$</th>
<th>$\tan\delta$, ($\times10^{-4}$)</th>
<th># of films</th>
</tr>
</thead>
<tbody>
<tr>
<td>5608.59</td>
<td>9400</td>
<td>0</td>
<td>empty</td>
<td>empty</td>
<td>0</td>
</tr>
<tr>
<td>5601.11</td>
<td>8000</td>
<td>0.100</td>
<td>3.19</td>
<td>49.1</td>
<td>1</td>
</tr>
<tr>
<td>5593.54</td>
<td>6890</td>
<td>0.201</td>
<td>3.20</td>
<td>50.2</td>
<td>2</td>
</tr>
<tr>
<td>5586.08</td>
<td>6080</td>
<td>0.303</td>
<td>3.20</td>
<td>50.3</td>
<td>3</td>
</tr>
<tr>
<td>5578.67</td>
<td>5480</td>
<td>0.406</td>
<td>3.20</td>
<td>49.4</td>
<td>4</td>
</tr>
<tr>
<td>5571.28</td>
<td>4480</td>
<td>0.511</td>
<td>3.19</td>
<td>49.5</td>
<td>5</td>
</tr>
<tr>
<td>5563.96</td>
<td>4970</td>
<td>0.616</td>
<td>3.18</td>
<td>50.6</td>
<td>6</td>
</tr>
</tbody>
</table>
The geometry of a split dielectric resonator and a picture of the resonator are shown in Figure 4a and b. The simplified schematic illustrates the main parts of the SPDR.

A pair of thin dielectric resonators and a metal enclosure of relatively small height are used in the construction of the SPDR fixture. This allows creating a strong evanescent electromagnetic field, not only in the air gap between the dielectric resonators, but also in the cavity region for radii greater than the radius of dielectric resonators. This simplifies numerical analysis and reduces possible radiation.
The measurement set-up shown in Figure 5 consists of the following components:

- PNA network analyzer, operating in appropriate frequency range for the fixture
- software, 85071E with option 300, installed in the PNA
- SPDR fixture

![Figure 5. Measurement setup](image-url)
SPDR Operation

SPDRs typically operate with the TE01d mode that has only an azimuthal electric field component, so the electric field remains continuous on the dielectric interfaces [2,3]. This makes the system insensitive to the presence of air gaps perpendicular to the z-axis of the fixture. The Rayleigh–Ritz method is used to compute the resonant frequencies, the unloaded Q-factors, and all other related parameters of the SPDR. The real part of permittivity \( \varepsilon' \) of the sample is found on the basis of measurements of the resonant frequencies and thickness of the sample as an iterative solution to equation (1).

\[
\varepsilon' = \frac{1 + f_0 - f_s}{hf_0 K_s(\varepsilon',h)}
\]  

(1)

Where \( h \) is the sample thickness, \( f_0 \) - the resonant frequency of the empty SPDR, \( f_s \) - the resonant frequency of the SPDR with the dielectric sample, \( K_s \) - a function of sample dielectric constant \( \varepsilon' \) and thickness \( h \). The function \( K_s \) is computed and tabulated for every specific SPDR. Exact resonant frequencies and the resulting values of \( K_s \) were computed for a number of \( \varepsilon' \) and \( h \) and tabulated. Interpolation has been used to compute \( K_s \) for any other values of \( \varepsilon' \) and \( h \). The initial value of \( K_s \) in the permittivity evaluation using formula (1) is taken to be the same as its corresponding value for a given \( h \) and \( \varepsilon' = 1 \).

Subsequent values of \( K_s \) are found for the subsequent dielectric constant values obtained in the iterative procedure. Because \( K_s \) is a slowly varying function of \( \varepsilon' \) and \( h \), the iterations using formula (1) converge rapidly.

The loss tangent is computed using equation (2)

\[
\tan \delta = \frac{Q^{-1} - Q_{DR}^{-1} - Q_c^{-1}}{pe}
\]

(2)

Where \( Q \) is the unloaded Q-factor of the resonant fixture containing the dielectric sample, and \( pe \) is the electric energy filling factor of the sample, defined as (3):

\[
pe = \frac{\iint_{V_s} \varepsilon_s EE^* dv}{\iint_{V_s} \varepsilon(v) EE^* dv} = h \varepsilon' K_1(\varepsilon',h)
\]

(3)

\( Q_c \) is the Q-factor depending on metal losses for the resonant fixture containing the sample (4):

\[
Q_c = \frac{\iint_{V} \mu_0 HH^* dv}{R_s \iint_{S} H_t H_t^*} = Q_{c0} K_2(\varepsilon',h)
\]

(4)

\( Q_{DR} \) is the Q-factor depending on metal losses for the empty resonant fixture, \( Q_{DR} \) is the Q-factor depending on dielectric losses in the dielectric resonators (5):

\[
Q_{DR} = \frac{Q_{DR0} f_0 p_{eDR}}{f_s p_{eDR}}
\]

(5)

\( p_{eDR} \cdot p_{eDR} \) - electric energy filling factors for the sample, and for the dielectric split resonator respectively. Similar to the function \( K_s \), \( K_1 \) and \( K_2 \) are functions of \( \varepsilon' \) and \( h \) that are computed and tabulated. Interpolation is used to compute the values of these functions for specific values of \( h \) and \( \varepsilon' \).
General

Typical uncertainty of the real permittivity is better than ±1 percent, providing that the thickness of a sample under test is measured with accuracy ±0.7 percent or better.
Typical loss tangent uncertainty: ±5 percent
Loss tangent resolution: $2 \times 10^{-5}$

Uncertainty of the real permittivity

The main source of uncertainty of the real permittivity is related to uncertainty of the thickness of the sample being tested. Relative error of real permittivity due to thickness uncertainty can be expressed as follows:

$$\frac{\Delta \varepsilon'}{\varepsilon'_r} = T \frac{\Delta h}{h}$$  \hspace{1cm} (6)

Where: $1 < T < 2$.

Usually the $T$ value is very close to unity except for thick, large permittivity samples. For such samples the value of $T$ increases, but always remains smaller than two. Additional factors affect the overall uncertainty, e.g. differences between real dimensions of the resonant fixture and permittivity of dielectric resonators, and the values assumed in computations. The most significant contribution to the overall $K_s$ error arises from coefficients related to the thickness and permittivity of the dielectric resonators. Assuming a given value for the thickness of the split post resonators and all other dimensions of the resonant structure, it is possible to choose a permittivity for the dielectric resonators such that we will get identical computed and measured resonant frequency values for an empty fixture. Exact numerical analysis has shown that in such a case $K_s$ errors due to uncertainty of dielectric resonator thickness and permittivity practically cancel out. If such an approach is used it is possible to compute $K_s$ coefficients for specific resonant structures with uncertainties better than 0.15 percent so one can estimate the total uncertainty for real permittivity as:

$$\frac{\Delta \varepsilon'}{\varepsilon'_r} \leq 0.15 + T \frac{\Delta h}{h}$$  \hspace{1cm} (7)

In principle, it is possible to further decrease systematic errors by 0.15 percent by making measurements of standard reference materials and introducing corrections of $K_s$ coefficients, but this would require perfectly machined specimens whose permittivity is defined with precision better than 0.15 percent.

Uncertainty of the loss tangent

Dielectric loss tangent uncertainty depends on many factors, mainly the Q-factor measurement uncertainty and the value of the electric energy filling factor. For a properly chosen sample thickness it is possible to resolve dielectric loss tangents to approximately $2 \times 10^{-5}$ for Q-factor measurements with an accuracy of about 1 percent.

NOTE: For very low loss materials, like sapphire or quartz, the measured Q-factor of the SPDR with a sample is often greater than the Q-factor for the empty SPDR. In spite of this, evaluated dielectric loss tangent values are greater than zero due to proper $Q_c$ and $Q_{DR}$ corrections.
Conclusions

The SPDR method of measuring the dielectric properties is best suited for nondestructive measurements of substrates and thin film materials. The method is simple, fast and offers superior accuracy.
References


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